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Free Convective MHD Oscillatory flow Past Parallel Plates in a Porous Medium with Heat Source and Chemical Reaction

J.Sasikumar, A.Govindarajan

Abstract— The effect of magnetic field on unsteady free convection oscillatory flow through infinite vertical parallel flat plates in a porous medium with heat source and chemical reaction, when free strem velocity, temperature and concentration oscillate in time about a non zero constant mean has been investigated. A cosed form analytical method is employed to solve the momentum and energy and concentration equations. Effects of various non dimensional parameters on velocity, temperature and concentration profiles have been analysed in detail and results shown in graphs.

Index Terms— MHD, Heat and Mass Diffusion, Free convection, Oscillatory, Porous, Heat source, Chemical Reaction

1 Introduction

The phenomenon of free convection arises in the fluid when temperature changes cause density variation leading to buoyancy forces acting on the fluid elements. This can be seen in our everyday life in the atmospheric flow, which is driven by temperature differences.

Free convective flow past vertical plate has been studied extensively by Ostrach [1-2] and many others. These studies are confined to steady flows only. In case of unsteady free convective flows Soundalgekar [3] studied the effects of viscous dissipation on the flow past an infinite verticalnporous plate. It was assumed that the plate temperature oscillates in such a way that its amplitude is small. The laminar free convection from a vertical plate has been investigated by Martynenko et al.[4] . Harris et al.[5] studied free convection from a vertical plate through a porous media. The natural convection flows adjacent to both vertical and horizontal surface, which result from the combined buoyancy effects of thermal and mass diffusion, was investigated by Gebhart and Pera [6] and Pera and Gabhart [7].

The flow problems of an electrically conducting fluid under the influence of magnetic field have attracted the interest of many authors in view of its applications to geophysics, astrophysics, engineering, and to the boundary layer control in the field of aerodynamics. On the other hand in view ofthe increasing technical applications using magneto hydrodynamics effect, it is desirable to extend many of the available viscous hydrodynamic solution to include the effects of magnetic field for those cases when the viscous fluid is electrically conducting. Rossow [8],Greenspan and Carrier [9] have studied extensively the hydromagnetic effects on the flow past a plate with or without injection/suction. The hydromagnetic channel flow and temperature field was investigated by Attia and Kotab[10]. Hossain et al. [11] have studied the MHD free convection flow when the surface kept at oscillating surface heat flux. The effect of fluctuating temperature and concentration on unsteady flow past plate with constant suction have been studied by Sharma et al. [12]. Sharma etal. [13] have studied the unsteady free convection oscillatory flow between parallel plates through porous medium assuming periodic temperature variations. Recently Sharma and Singh [14] also studied the effect of magnetic field and thermal diffusion on oscillatory free convection flow past plate with viscous heating. Sekhar et al (2012) studied the unsteady MHD mixed convective oscillatory flow of an electrically conducting optically thin fluid flow though a planar channel filled with saturated porous medium. The effect of buoyancy, heat source, thermal radiation and chemical reaction are taken into account embedded with slip boundary condition, varying temperature and concentration.

The fundamental concept behind MHD is that magnetic fields can induce currents in a moving conductive fluid,which in turn creates forces on the fluid and also changes the magnetic field itself. The set of equations which describe MHD are a combination of the Navier-Stokes equations of fluid dynamics and Maxwell's equations of electromagnetism. MHD applies quite well to astrophysics,content of the universe is made up of plasma, including stars, the interplanetary medium (space between the planets),the interstellar medium (space between the stars), the intergalactic medium, nebulae and jets. Sunspots are caused by the Sun's magnetic fields, the solar wind is also governed by MHD.

- J.Sasikumar, SRM University, India . E-mail: sasikumar.j@ktr.srmuniv.ac.in
- A. Govindarajan, SRM University, Kattankulathur, Chennai, India. E-mail: govindarajan.a@ktr.srmuniv.ac.in
- E.P.Siva, SRM University, Kattankulathur, Chennai, India. E-mail: siva.e@ktr.srmuniv.ac.in

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MHD is receiving considerable attention due to the possibilities it offers for much higher thermal efficiencies in power plants. Mass diffusion rates can changed tremendously with chemical reactions. The chemical reaction effects depend whether the reaction is homogeneous or heterogeneous. This depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems, the reaction is heterogeneous, if it takes place at an interface and homogeneous, if it take place in solution. In majority cases, a chemical reaction depends on the concentration the concentration of the species itself. A reaction is said to be first order, if the rate of reaction is directly proportional to the concentration itself . A few representative areas of interest in which heat and mass transfer combined along with chemical reaction play an important role in chemical industries like in food processing and polymer production. Chambre and Young have analyzed a first order chemical reaction in the neighborhoodof a horizontal plate. Das et al. (7) have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started vertical plate with uniform heat flux and mass transfer.An analysis of first order homogeneous chemical reaction and heat source on MHD oscillatory flow of visco – elastic fluid through a channel filled with saturated porous medium are reported by Devika et al (2013).Boundary layer flows of fluids of small electrical conductivity with the effect of simultaneous thermal and mass diffusion in oscillatory flow are important in the field of many engineering applications.
Therefore the object of the present paper is to study the effects of the magnetic field on the flow of a viscous, incompressible and electrically conducting fluid between two vertical parallel porous plates in the presence of heat source and chemical reaction with oscillating free stream velocity assuming that temperature and concentration is also oscillating with time about a non zero constant mean.

2 Problem Formulation

We consider the unsteady Couette flow of a viscous incompressible electric conducting fluid through a highly porous medium bounded between two infinite vertical porous plates. One of which is suddenly moved from rest with a free stream velocity that oscillate in time about a constant mean. We take x*-axis along the moving vertical plate in the vertically upward direction and y*-axis is taken normal to this plate. The other stationary vertical plate is assumed to be situated at y* = b at temperature T_b.
consider the free-stream velocity distribution of the form:

$$U^* = U_0\left(1+\varepsilon e^{i\omega^* t^*}\right)$$

where U_0 is the mean constant free-stream velocity, ω* is the frequency of oscillations and t* is the time.
The basic equation of magneto fluid dynamics and conventional fluid dynamics are different by only additional force term due to electromagnetic field. The Maxwell's equations have to be satisfied in the entire field.
In order to derive the basic equations for the problem under consideration, the following assumptions are made:

1. The flow is unsteady, laminar and the magnetic field is applied perpendicular to the plates.
2. The fluid is viscous incompressible and finitely conducting with constant physical properties.
3. The magnetic Reynolds number is taken to be small enough so that the induced magnetic field is neglected.
4. Hall Effect, electrical and polarization effects are neglected.

The governing equations of the flow are:

$$\frac{\partial u^*}{\partial t^*} = \frac{\partial U^*}{\partial t^*} + \gamma \frac{\partial^2 u^*}{\partial y^{*2}} + g\beta(T^* - T_b^*) + g\beta_c(C^* - c_b^*) - \left(\frac{\bar{J} X \bar{B}}{\rho}\right) - \frac{\gamma u^*}{k^*}$$

Where

$\bar{J} X \bar{B} = \sigma(\bar{\gamma} X \bar{B}) X \bar{B}$; $\left(\frac{\bar{J} X \bar{B}}{\rho}\right) = \frac{(u^* - U^*)\sigma B^2}{\rho}$ and we get

$$\frac{\partial u^*}{\partial t^*} = \frac{\partial U^*}{\partial t^*} + \gamma \frac{\partial^2 u^*}{\partial y^{*2}} + g\beta(T^* - T_b^*) + g\beta_c(C^* - c_b^*) - \left(\frac{(u^* - U^*)\sigma B^2}{\rho}\right) - \frac{\gamma u^*}{k^*} \quad (1)$$

$$\frac{\partial T^*}{\partial t^*} = \alpha \frac{\partial^2 T^*}{\partial y^{*2}} - Q(T^* - T_b^*) \quad (2)$$

$$\frac{\partial C^*}{\partial t^*} = D \frac{\partial^2 C^*}{\partial y^{*2}} - K(C^* - C_b^*) \quad (3)$$

The boundary conditions are

$$y^* = 0;\ u^* = U_0\left(1 + \epsilon e^{i\omega^* t^*}\right) \quad T^* = T_0^* + \epsilon(T_0^* - T_b^*)e^{i\omega^* t^*}$$

$$C^* = C_0^* + \epsilon(C_0^* - C_b^*)e^{i\omega^* t^*}$$

$$y^* = b;\ u^* = 0$$

$$T^* = T_b^* ;\ C^* = C_b^*$$

$$y = \frac{y^*}{b} ;\quad u = \frac{u^*}{U_0} ;\quad U = \frac{U^*}{U_0} ;\quad t = \omega^* t^* ;\quad \omega = \frac{\omega^* b^*}{\gamma}$$

$$\theta = (T^* - T_b^*)/(T_0^* - T_b^*) ;\quad G_r = \frac{g\beta b^2(T_0^* - T_b^*)}{\gamma U_0} ;$$

$$G_c = \frac{g\beta_c b^2(C_0^* - C_b^*)}{\gamma U_0} ;\quad C = (C^* - C_b^*)/(C_0^* - C_b^*) ;$$

$$S_c = \frac{\gamma}{D} ;\quad M = \sqrt{\frac{\sigma B^2 b^2}{\rho\gamma}} ;\quad P_r = \frac{\gamma}{\alpha} ;$$

From equations (1) , (2) , and (3) we get
The momentum equation as

$$w\left(\frac{\partial u}{\partial t} - \frac{\partial U}{\partial t}\right) = \frac{\partial^2 u}{\partial y^2} + Gr\,\theta + Gc\,C - (s^2 + M^2)u + M^2 U$$

The energy equation as

$$\mathbf{P_r}\, w \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} - Q\frac{b^2}{\alpha}\theta$$

The concentration equation as

$$\omega\left(\frac{\partial C}{\partial t}\right) = \frac{1}{S_c}\left(\frac{\partial^2 C}{\partial y^2}\right) - \left(\frac{b^2 U_0}{\gamma^2}\right) K_c C$$

with the boundary conditions

$$\left.\begin{array}{c} y = 0, u = 1 + \varepsilon e^{it} = \theta = C \\ y = 1, u = \theta = C = 0 \end{array}\right\}$$

3 Solution of the Problem

Since the amplitudes of the free-stream velocity, temperature and concentration variation ε(<<1) is very small, we assume the solutions in the following form :

$$u(y,t) = u_0(y) + \varepsilon\, u_1(y)e^{it} \quad (8)$$

$$\theta(y,t) = \theta_0(y) + \varepsilon\, \theta_1(y)e^{it} \quad (9)$$

$$C(y,t) = C_0(y) + \varepsilon\, C_1(y)e^{it} \quad (10)$$

and for free stream velocity

$$U = 1 + \varepsilon e^{it} \quad (11)$$

Substituting (9) in (5) and (10) in (6)

$$-P_r\omega i\theta_1 + \theta_1'' - \left(\frac{Qb^2}{\alpha}\right)\theta_1 = 0 \;=>\; \theta_1'' - \left(\frac{Qb^2}{\alpha} + iP_r\omega\right)\theta_1 = 0 \quad (12)$$

$$\left(\theta_0'' - \frac{Qb^2}{\alpha}\theta_0\right) = 0 \quad (13)$$

$$C_1'' - \left\{\left(\frac{b^2U_0}{\gamma^2}\right)K_cS_c - i\omega S_c\right\}C_1 = 0 \quad (14)$$

$$\frac{1}{S_c}C_0'' - \left(\frac{b^2U_0}{\gamma^2}K_c\right)C_0 = 0 \;=>\; C_0'' - \left(\frac{b^2U_0}{\gamma^2}\right)S_c.K_cC_0 = 0 \quad (15)$$

From (4)

$$u_1'' - (s^2 + M^2 + i\omega)u_1 = -G_r\theta_1 - G_cC_1 + (i\omega + M^2) \quad (16)$$

$$u_0'' - (s^2 + M^2)u_0 = -(G_r\theta_0 + G_cC_0 + M^2) \quad (17)$$

With the corresponding boundary conditions

$$y = 0, u_0 = 1, u_1 = 1, \theta_0 = 1, \theta_1 = 1, C_0 = 1, C_1 = 1$$

$$y = 1, u_0 = 0, u_1 = 0, \theta_0 = 0, \theta_1 = 0, C_0 = 0, C_1 = 0$$

Solving (12) & (13)

$$\theta_1 = a_{11}e^{n_2y} + a_{12}e^{-n_2y} \;;\; n_2^2 = \left(\frac{Qb^2}{\alpha} + iP_r\omega\right)$$

$$\theta_0 = a_{01}e^{n_3y} + a_{02}e^{-n_3y} \;;\; n_3^2 = \left(\frac{Qb^2}{\alpha}\right)$$

Solving (14) & (15) ,

$$n_6^2 = \{S^2 + M^2 + i\omega\}$$

$$\therefore u_1 = (c_{11}e^{n_6y} + c_{12}e^{-n_6y}) - \frac{G_r\theta_1}{(n_2^2 - n_6^2)} - \frac{G_cC_1}{(n_4^2 - n_6^2)} + \left(\frac{M^2 + i\omega}{s^2 + M^2 + i\omega}\right)$$

Solving (17),

$$n_7^2 = (s^2 + M^2)$$

$$u_0 = (c_{01}e^{n_7y} + c_{02}e^{-n_7y}) - \frac{G_r\theta_0}{(n_3^2 - n_7^2)} - \frac{G_cC_0}{(n_5^2 - n_7^2)} + \left(\frac{M^2}{n_7^2}\right)$$

$$a_{01} = \frac{-e^{-2n_3}}{1-e^{-2n_3}} \quad a_{02} = \frac{1}{1-e^{-2n_3}} \quad a_{11} = \frac{-e^{-2n_2}}{1-e^{-2n_2}} \;;\; \;;$$

$$a_{12} = \frac{1}{1-e^{-2n_2}}$$

$$b_{01} = \frac{-e^{-2n_5}}{1-e^{-2n_5}} \;;\; b_{02} = \frac{1}{1-e^{-2n_5}} \;;\; b_{11} = \frac{-e^{-2n_4}}{1-e^{-2n_4}} \;;\; b_{12} = \frac{1}{1-e^{-2n_4}}$$

$$c_{01} = \left\{1 + \frac{G_r}{n_3^2 - n_7^2} + \frac{G_c}{n_5^2 - n_7^2} - \frac{M^2}{n_7^2}\right\}\left(1 - \frac{e^{n_7}}{A}\right) - \frac{M^2}{An_7^2} \;;$$

$$c_{02} = \left\{1 + \frac{G_r}{n_3^2 - n_7^2} + \frac{G_c}{n_5^2 - n_7^2} - \frac{M^2}{n_7^2}\right\}\left(\frac{e^{n_7}}{A}\right) + \frac{M^2}{An_7^2}$$

$$c_{11} = \left\{1 + \frac{G_r}{n_2^2 - n_6^2} + \frac{G_c}{n_4^2 - n_6^2} - \frac{M^2 + i\omega}{n_6^2}\right\}\left(1 - \frac{e^{n_6}}{B}\right) - \frac{M^2 + i\omega}{Bn_6^2} \;;$$

$$c_{12} = \left\{1 + \frac{G_r}{n_2^2 - n_6^2} + \frac{G_c}{n_4^2 - n_6^2} \frac{M^2 + i\omega}{n_6^2}\right\}\left(\frac{e^{n_6}}{B}\right) + \frac{M^2 + i\omega}{Bn_6^2} \;;$$

4 Results and Discussion

The effect of magnetic field and convection on transient velocity, transient temperature and concentration when the plate is subjected to oscillatory velocity, temperature and concentration are analysed. Numerical calculation are carried out by drawing graphs for velocity, temperature and concentration with different values of the Grashoff number Gr, the modified Grashoff number Gc, the Prandtl number Pr,the Schmidt number Sc, the frequency ω and the Hartmann number M.

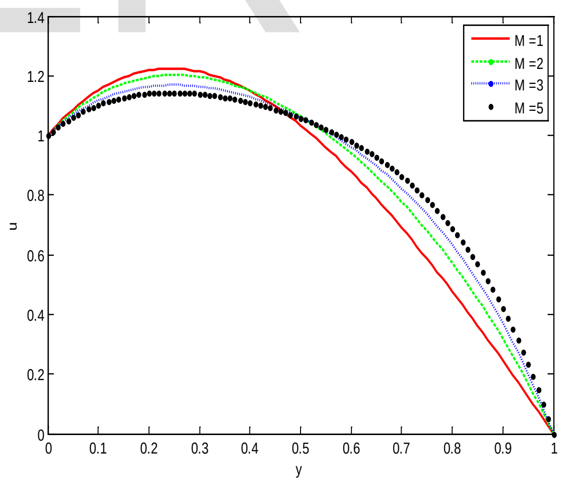

Fig.1 velocity profiles for different values of M.

Fig.1 shows that Velocity decreases as M increases and after middle of the channel velocity increases. Also velocity increases as the values of Grashoff number Gr and Gc increases.

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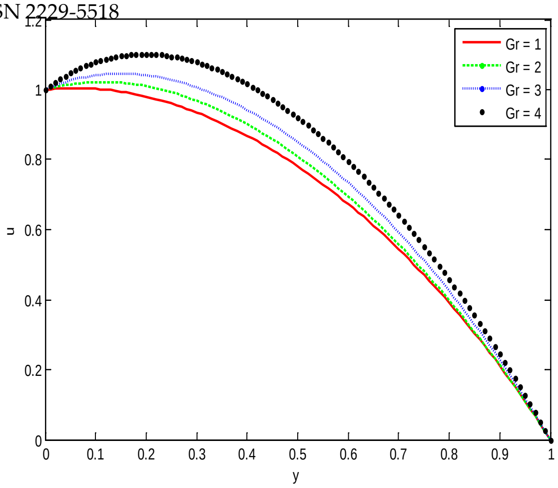

Fig.2 velocity profiles for different values of Gr.

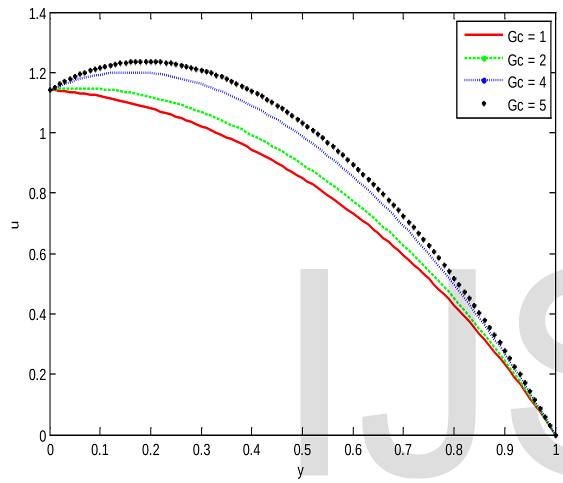

Fig.3 velocity profiles for different values of Gc.

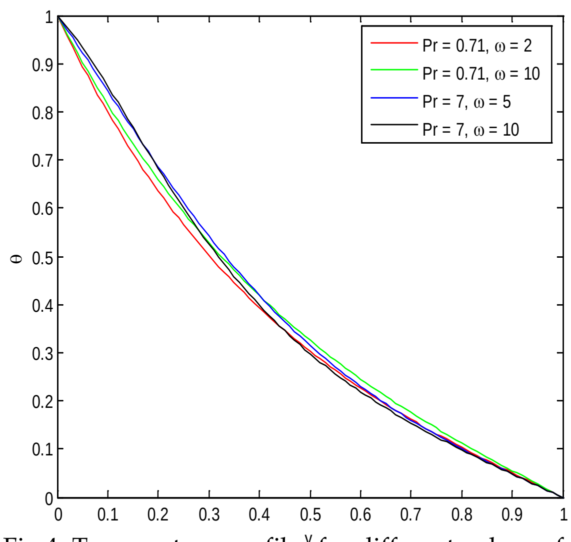

Fig.4 Temperature profiles for different values of Pr and ω.

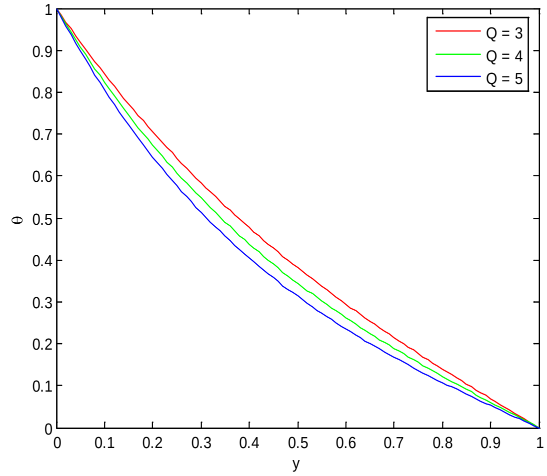

Fig.5 Temperature profiles for different values of Q

Fig 4 and 5 shows that temperature oscillates when Pr and ω increases and temperature decreases as the parameter Q increases.

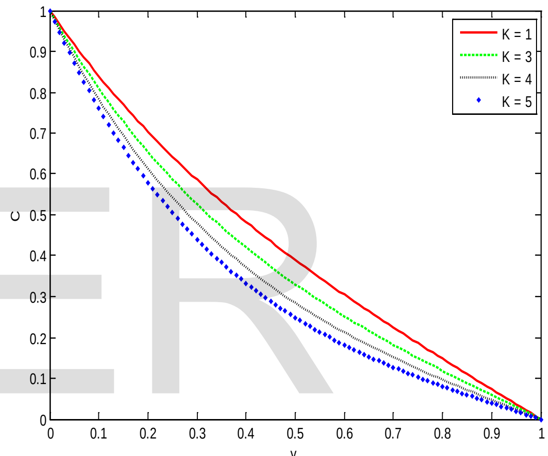

Fig.6 Concentration profiles for different values of K .

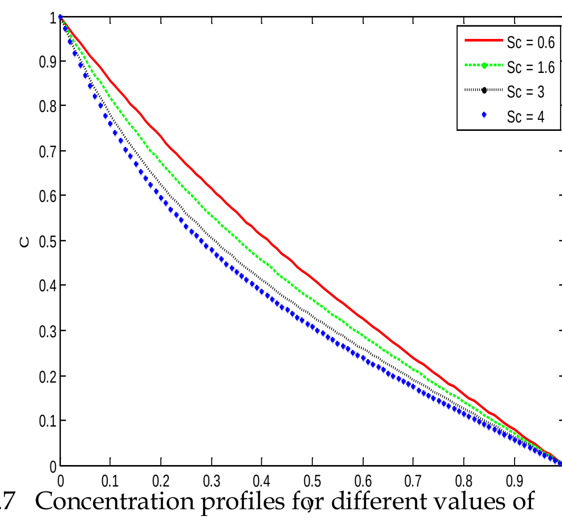

Fig.7 Concentration profiles for different values of Sc .

From fig.6 and fig.7 , transcient concentration decreases as the values of chemical reaction parameter K and Schmidt number Sc increases.

5 Nomenclature

C = dimensionless species concentration,
C_0^* = species concentration at plate (y=0)
C_0 = dimensionless species mean concentration,
C_b^* = species concentration at plate (y=b)
D = chemical molecular species,
P_r= Prandtl number
Q_w = constant heat flux per unit area,
M = Harmann number,
S_c = Schmidt number,
t = time,
T_0 = mean temperature of the fluid,
G = acceleration due to gravity,
G_r = thermal Grashof number,
G_c = modified(solutal) Grashof number,
N_u = Nusselt number,
U = dimensionless free stream velocity,
u^* = component of the velocity,
u = dimensionless velocity component,
x^* ,y^* = Cartesian co-ordinates,
x,y = dimensionless Cartesian co-ordinates,
Greek symbols
α = thermal diffusivity,
β = volumetric coefficient of thermal expansion,
β_c = volumetric coefficient of thermal expansion with concentration,
T_0^* = temperature in free stream,
T_b^* = temperature at the plate,
U^* = free stream velocity component,
σ = magnetic permeability,
ε = amplitude of suction velocity (<<1),
κ = thermal conductivity,
ν = kinematics viscosity,
θ = dimensionless temperature,
ρ = density,
τ = skin-friction at the plate,
τ_m = mean skin-friction at the plate,
ω = frequency of oscillations.

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